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An Information Entropy Approach to the Small-Lot Concept

Boaz Ronen and Roni Karp

***Abstract*—Just-In-Time and Total Quality Management advocate the use of small lots in production, as they yield higher throughput, better quality, lower response time, less operating expenses, better due date performance, and less work in process. Implementation of the small-lot concept is often resisted by MIS managers and production people who feel that the larger number of lots—the apparently inevitable result of reducing lot size, will entail more information and stronger information technology. By developing a normative model, based on the entropy measurement, this paper claims that the move toward smaller lots implies less information needs. Theorems concerning the relationship between quantity of information and lot size are proved, showing that the new directions that manufacturing is taking entail less information needs.**

I. Introduction

The VP for Operations of a large electronics company is willing to move toward the use of smaller lot sizes in the production process. Specifically, he is willing to cut the batch size by almost half. From the theoretical standpoint, on the basis of the practical experience of many firms, he estimates that this step will increase productivity, shorten response time, and yield better quality. Before embarking on this course of action he has to cope with the resistance to change of the VP for MIS and the floor foremen. They insist on getting better support and more information to handle the increased number of information items that will be created by increasing the number of lots. They claim that in the past they produced 500 batches per month, that the new procedure will mean about 1000 batches per month, and that the existing system will not be able to handle this double load.

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The purpose of this paper is to prove that by using the small-lot concept, less information items will be needed than before. There is no need for major investments in information technology in adopting the small-lot concept.

The last two decades have witnessed the emergence of new management philosophies and techniques which, when implemented, completely change the manufacturing paradigms and practices. Among the emerging philosophies, the leaders [3] are *just in time* (JIT) [6], [7], *optimized production technology* (OPT) [5], *total quality management* (TQM) [2], and the *theory of constraints* (TOC) [8].

Theses methods of management have brought on changes in the approach to production lot sizes. Because of the high set-up costs, past production management theories favored large-lot production. Now, with the advent of the new production management theories, more and more firms are producing in smaller lots.

The subject is particularly interesting, since small-lot production should intuitively require a stronger information system than large-lot production, due to the larger number of lots there are to manage. It is the intention of this article to show that a transition to small-lot production brings with it, a decrease in information needs, and not an increase as might be thought.

The paper copes with the problem by using the entropy measurement to measure quantity of information, and constructing a model to evaluate the information needs and the effect of the lot size on the quantity of required information. The next section briefly presents entropy as a measure of information. In Section III we develop an analytical model, based on the entropy measure, and prove that reducing lot sizes causes a lessening of information needs. Section IV concludes the discussion.

II. Entropy as a Measure of Information Quantity

One of the means of measuring the quantity of information is Shannon's (1948) entropy function. The definition of the entropy function is as follows:

Given a group of events $E = \{e_1, \cdots, e_n\}$ and the *a priori* probabilities of the events' occurrence $P = \{p_1, \cdots, p_n\}$, where $p_i \geq 0$ and

$$\sum_{i=1}^{n} p = 1$$

the entropy function of E is defined as follows:

$$H = -K \sum_{i=1}^{n} p_i \cdot \log(p_i)$$

where $0 \cdot \log(0) = 0$. For simplicity, and without losing generality, we will use $K = 1$. For more insight into the entropy approach and its applications, the reader may refer to [1, ch. 3] and [4].

III. The Information Requirements of Small-Lot Production

A. *The Information Paradox of Small-Lot Manufacturing*

Production managers considering a move to small-lot production usually react by requesting an upgrading of their information system. The rationale for the above request is as follows:

1) Lot size is inversely proportional to the number of lots.

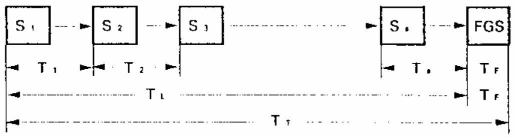

Fig. 1 Times in the entropy model.

2) The amount of information is directly proportional to the number of lots: $II = C1 \cdot NL$. 1) and 2) imply:
3) The amount of information needed is inversely proportional to the size of the lot: $II = C2 \cdot (1/LS)$ where: II = number of information items needed, NL = number of lots, LS = lot size, and $C1$ and $C2$ are constants.

B. The Entropy Model

Description: In this model entropy measures the level of information needed in order to ascertain the location of a lot along the assembly line, when the probability of its being at any one station is known. Being "at a station" means either in or before the station.

We will check the entropy of a production line, and see how it varies when the lot size of a certain product is changed, while the total production level remains constant.

The higher the uncertainty in the system, the higher the entropy, and the more information is required to understand what is happening in it. The analysis will be as follows:

1) Calculate the entropy of a single lot.
2) Calculate the amount of information required concerning the production line.

The Entropy of a Production Line The requirement is to produce a given quantity P of products. The line is serial, containing S stations. Let us take a time unit T_T, greater than or equal to the time it takes to produce the required amount, and relate it to the frequency of the lot being at any one station. Figure 1 contains a schematic representation of the line, where

S_i	station i
S	number of stations
T_i	time needed to process a lot in station i. For simplicity, we consider T_i as deterministic.
FGS	finished goods storage
T_F	finished goods storage time
T_L	time the lot has spent in the production line
T_T	total time the lot has been in the system

The probability of its being at that station (PR_i) is given by

$$PR_i = T_i/T_T.$$

The lot's lead time is given by

$$T_L = \sum_{i=1}^{S} T_i.$$

The probability of its being anywhere along the line is

$$PR_L = T_L/T_T = \sum_{i=1}^{S} T_i/T_T.$$

The rest of the time the lot is in the finished goods deposit, or it is already with the customer. This time is given by

$$T_F = T_T - T_L = T_T - \sum_{i=1}^{S} T_i.$$

Note: This can also be the time spent by the lot before production, i.e., as raw material, without affecting the model. Raw material can be considered as any other station. The probability of the lot being in the finished goods deposit is given by

$$PR_F = T_F/T_T = (T_T - \sum_{i=1}^{S} T_i)/T_T.$$

The lot's entropy $H(B)$ is

$$H(B) = -\sum_{i=1}^{S}(PR_i \cdot \log(PR_i)) - PR_F \cdot \log(PR_F).$$

We will now check the amount of data which needs to be transmitted from the line (the system's entropy).

We claim that it is necessary to transmit data only from lots located in the assembly line itself. In other words, information about lots in the finished goods deposit is not needed to manage the line and is therefore irrelevant. Hence, to obtain the amount of data required from the line (the system's entropy), we need only multiply a single lot's entropy by the number of lots and by the relative frequency of its being in the line. Let B = number of lots. The system's entropy $H(S)$ will be given by

$$H(S) = B \cdot PR_L \cdot H(B) = B \cdot (\sum_{i=1}^{S} T_i/T_T) \cdot H(B).$$

Let us check the entropy of a line with S identical stations. Let P = number of items, S = number of stations, N = number of items per lot, T = lead time of one item per station. Thus, $N \cdot B = P$. Since the stations are identical, the lead time of a lot in one station is $T_i = N \cdot T$ for all i. The total lead time is given by

$$T_L = \sum_{i=1}^{S} T_i = S \cdot N \cdot T.$$

Without loss of generality, assume the time reference unitT_T to be a multiple of the longest lead time. This was chosen arbitrarily. It is the time it would take to produce the whole amount if it were to be produced in one lot:

$$T_T = C \cdot (T_L)_{B=1} = C \cdot (T_L)_{P=N} = P \cdot T \cdot S \cdot C$$

The meaning of the constant C is as follows: when the entire amount is produced in a single lot, C is the ratio between the gross time (process lead time + time spent in finished goods deposit) and the net time (process lead time only). By that we assume a linear relationship between the gross time and the net time. This is often done in the literature (see [6] or [8]). Hence $C \geq 1$. The probability of a lot being in the ith station is

$$PR_i = T_i/T_T = N \cdot T/T_T = N \cdot T/(P \cdot T \cdot S \cdot C) = N/(P \cdot S \cdot C).$$

The probability of its being anywhere along the line is

$$PR_L = \sum_{i=1}^{S} PR_i = S \cdot PR_i = S \cdot N/(P \cdot S \cdot C) = N/(P \cdot C).$$

The time spent by the lot in the finished goods deposit is

$$T_F = T_T - T_L = T_T - \sum_{i=1}^{S} T_i$$
$$= P \cdot T \cdot S \cdot C - S \cdot N \cdot T = S \cdot T \cdot (P \cdot C - N).$$

The probability of its being in the finished goods deposit is

$$PR_F = T_F/T_T = S \cdot T \cdot (P \cdot C - N)/(S \cdot T \cdot P \cdot C)$$
$$= (P \cdot C - N)/(P \cdot C).$$

The lot's entropy as a function of its size is

$$H(B) = -\sum_{i=1}^{S}(PR_i \cdot \log(PR_i)) - PR_F \cdot \log(PR_F)$$
$$= -\frac{N}{P \cdot C} \cdot \log \frac{N}{P \cdot S \cdot C} - \frac{P \cdot C - N}{P \cdot C} \cdot \log \frac{P \cdot C - N}{P \cdot C}.$$

Substituting $B = P/N$ we get the lot's entropy as a function of the number of lots:

$$H(B) = -\frac{1}{B \cdot C} \cdot \log \frac{1}{B \cdot S \cdot C} - \frac{B \cdot C - 1}{B \cdot C} \cdot \log \frac{B \cdot C - 1}{B \cdot C}$$

The system's entropy as a function of the number of lots will be

$$H(S) = B \cdot PR_L \cdot H(B) = B \cdot (N/(P \cdot C)) \cdot H(B).$$

Since $B = P/N$ we obtain

$$H(S) = B \cdot (N/(P \cdot C)) \cdot H(B) = (1/C) \cdot H(B)$$
$$= -\frac{1}{C^2 \cdot B} \cdot \log \frac{1}{B \cdot S \cdot C} - \frac{B \cdot C - 1}{C^2 \cdot B} \cdot \log \frac{B \cdot C - 1}{B \cdot C}.$$

The system's entropy as a function of the lot size will be given by

$$H(S) = -\frac{N}{C^2 \cdot P} \cdot \log \frac{N}{P \cdot S \cdot C} - \left(\frac{1}{C} - \frac{N}{P \cdot C^2}\right) \cdot \log \left(1 - \frac{N}{P \cdot C}\right).$$

Let us now check the system's entropy as a function of the number of lots. The entropy of the previously defined line (serial, with all production times equal) is a function of the number of lots, the number of stations and the time reference chosen (this comes across in the constant C as explained before).

Theorem 1: For $B \geq 2$ and $S \geq 2$, entropy of the system as a function of the number of lots is monotonically decreasing.

Proof: Let H_1 be a continuous function identical to the system entropy function $H(S)$ except that it is continuous in B. For convenience we will check the two members separately. Their derivatives will be denoted by D_1 and D_2, respectively. "log" will denote a logarithm with base 2, "ln" will denote the natural logarithm. Now $dH_1/dB = D_1 + D_2$.

Checking the first member we get

$$D_1 = -\frac{1}{\ln(2)} \cdot \frac{1}{C^2} \cdot \left\{-\frac{1}{B^2} \cdot \ln \frac{1}{B \cdot S \cdot C} + \frac{1}{B} \cdot \frac{B \cdot S \cdot C}{1} \cdot \left(-\frac{1}{B^2 \cdot S \cdot C}\right)\right\}$$
$$= +\frac{1}{\ln(2)} \cdot \frac{1}{C^2} \cdot \left(\frac{1}{B^2} \cdot \ln \frac{1}{B \cdot S \cdot C} + \frac{1}{B^2}\right)$$
$$= +\frac{1}{\ln(2)} \cdot \frac{1}{C^2} \cdot \frac{1}{B^2} \cdot \left(\ln \frac{1}{B \cdot S \cdot C} + 1\right).$$

The derivative is zero when $\ln(1/(B \cdot S \cdot C)) = -1$. Hence the first member's optimum is achieved at $B = e/(S \cdot C)$. The first member has one optimum. Since $C \geq 1$ by definition and $S \geq 2$ by the theorem's condition, at the optimum $B \leq e/2 < 1.36$. A further derivation will show that this is indeed a maximum. Therefore the member is monotonically decreasing for $B \geq 2$.

Checking the second member we get its derivative, D_2:

$$D_2 = -\frac{1}{\ln(2)} \cdot \frac{1}{C} \cdot \left\{\frac{1}{C \cdot B^2} \cdot \ln \left(1 - \frac{1}{B \cdot C}\right)\left(1 - \frac{1}{B \cdot C}\right) \cdot \frac{B \cdot C}{B \cdot C - 1} \cdot \frac{1}{B^2 \cdot C}\right\}$$
$$= -\frac{1}{\ln(2)} \cdot \frac{1}{C^2 \cdot B^2} \cdot \left\{\ln \left[1 - \frac{1}{B \cdot C}\right] + 1\right\}$$

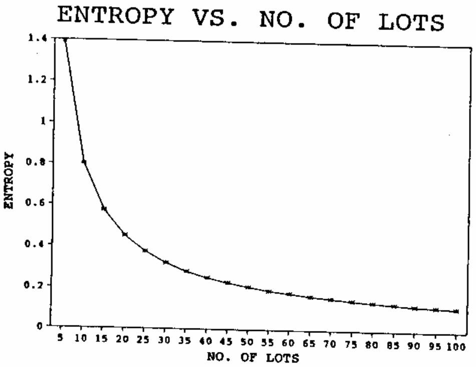

Fig. 2. Entropy as a function of the number of lots.

The derivative is zero when $\ln((B \cdot C - 1)/(B \cdot C)) = -1$, or$B = 1/C \cdot (1 - e^{-1})$. This member also has one optimum, which a further derivation will show to be a maximum. Since by definition $C \geq 1$, at the maximum we have $B \leq 1/(1 - e^{-1}) < 1.582$. We saw that both members have maxima at points between 1 and 2, and these are their only optimum points. Hence the function as a whole is monotonically decreasing in B for $B \geq 2$. The transition from the continuous function $H1$ to the discontinuous entropy function $H(S)$ is immediate. Q.E.D.

Implications:

1.) The condition $S \geq 2$ does not limit the theorem's generality, since any line has at least two stations. Anything less than that is not considered a production line.
2.) An increase in the number of lots (or a decrease in the size of lots) reduces entropy.
3.) There is a special situation in which entropy increases along with the number of lots. This happens when we go from one to two lots, and when the time of reference implies that the lot is in the finished goods deposit for a very short time (i.e., C is close to 1).

Theorem 2: As the number of lots tends to infinity, entropy tends to zero.

Proof: When the number of lots is large, we get $(B \cdot C - 1)/(B \cdot C) \to 1$, hence $\log((B \cdot C - 1)/(B \cdot C)) \to 0$. In other words, the second member tends to zero. We know as well that

$$\lim_{A \to 0} A \cdot \log(A) = 0$$
$$-\frac{1}{C^2 \cdot B} \cdot \log \frac{1}{B \cdot S \cdot C} = -\frac{S}{C} \cdot \frac{1}{B \cdot S \cdot C} \cdot \log \frac{1}{B \cdot S \cdot C} \to 0.$$

Hence the first member also tends to zero as the number of lots tends to infinity. Therefore the entire function tends to zero as the number of lots tends to infinity. Q.E.D.

Note: there will be no need for information from the line if the number of lots becomes very large. This is the case where the system will either need no control or will be controlled by itself.

Evaluation of the function for a line with 10 stations: For purposes of illustration, the entropy for an assembly line of 10 stations was computed as a function of the number of lots ($C = 1$). Figures 2 and 3 show entropy as a function of the number of lots and their size.

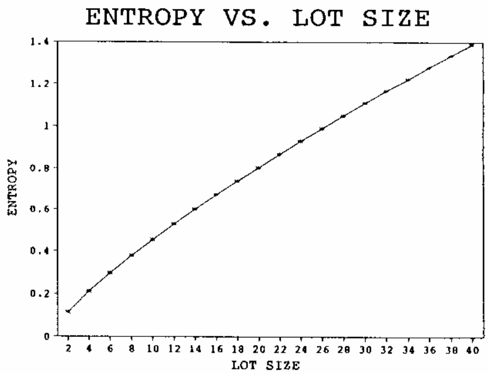

Fig. 3. Entropy as a function of lot size.

IV. Conclusions

As the above results show, the system's entropy decreases as the number of lots rises. The practical significance of this result is that a reduction in lot size brings about a reduction in the information needs of the assembly line manager. This result shows that the information system might theoretically be cut back along with a reduction in lot sizes. The widely held belief that production in smaller lots requires a stronger information system causes resistance to change and delays its adoption by manufacturing firms. The present study disproves this belief. We show that a reduction in lot size reduces entropy, i.e., it reduces uncertainty. This means that less information is needed in order to manage the assembly line. This research was based on the assumption that the processing time at a given production station includes the transportation time and waiting times. The relaxation of this assumption may need a more complicated treatment and may yield similar results. Further research is suggested on this important issue, especially empirical field results that will show these results in real-life conditions.

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